647510 **BSc5MatC503x** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination

Oct/Nov - 2023 (NEW COURSE)

Boolean Algebra & Complex Analysis -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two. (10)

1. R is an equivalence relation on set X then prove that
   (i) $\cup_{x \in X}[x] = X$
   (ii) xRy $\Leftrightarrow [x] = [y]$
2. In usual notation prove that
   $(a * b) * c = a * (b * c)$ and $(a \oplus b) \oplus c = a \oplus (b \oplus c)$
3. Prove that every chain is a distributive lattice.

Que.1(B) Answer any one. (04)

1. In usual notation show that (S_{30}, D) is poset.
2. Define Cover, Draw Hasse diagram of (S_{30}, D).

Que.2(A) Answer any two. (10)

1. If S = {a, b, c}, B = {0, 1} then show that f: P(S) → B is Boolean homomorphism for (P(S), ∩, ∪, ^{c}, ϕ, S) and (B, ∗, ⊕, ', 0, 1) where
   $f(x) = 1, b \in X$
   $f(x) = 0, b \notin X.$
2. If (B, ∗, ⊕, ', 0, 1) is Boolean algebra and $x_1, x_2 \in B$ then prove that
   $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$
3. Define Minterm and Maxterm, Obtain sum of product canonical form of
   $\alpha(x_1, x_2, x_3) = x_1 x_3 \oplus x_2 x_3{}'$

Que.2(B) Answer any one. (04)

1. If (B, ∗, ⊕, ', 0, 1) is finite Boolean algebra and $x \in B$, $x \neq 0$ then there exist an atom a in B such that $a \leq x$.
2. For Boolean statement prove that
   $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c') = b' * (a \oplus c)$

Que.3(A) Answer any two. (10)

1. In usual notation obtain Cauchy – Riemann conditions in Cartesian form.
2. Prove that $f(z) = \frac{\overline{(z)}^2}{z}$, $z \neq 0$
   and f (z) = 0 , z = 0 satisfied C – R conditions at origin however f(z) is not analytic at origin.
3. Obtain Laplace equation in polar form.

Que.3(B) Answer any one. (04)

1. Prove that sinz is an analytic function.
2. Find an analytic function f (z) = v + i v such that u – v = e^{-x}(sin y + cos y)

Que.4(A) Answer any two. (10)

1. State and prove Cauchy's fundamental theorem for integration.
2. Find $\int_C \frac{dz}{z-\frac{2}{z}}$; where C: $|z-1| = 2$.
3. Show that $\left|\int_C \frac{z+4}{z^3-1}\, dz\right| \leq \frac{6\pi}{7}$ Where C be the arc of the circle $|z| = 2$ from z = 2 to z = 2i.

Que.4(B) Answer any one. (04)

1. Find: $\int_C \frac{dz}{(z-3)}$ where C: $|z - 2| = 5$.
2. Find: $\int_C \frac{(5z-2)dz}{z(z-3)}$ where C: $|z + 1| = \frac{3}{2}$.

Que.5(A) Answer any two. (10)

1. State and prove Morera's theorem.
2. State and prove Liouville's theorem.
3. Find: $\int_C \frac{z^2+3}{z^2(z-4)}$ dz where C: $|z| = 1$.

Que.5(B) Answer any one. (04)

1. Find: $\int_C \frac{5z^2-7z+3}{(z-1)^2}$ dz where C: $|z| = 2$.
2. In usual notation state and prove Cauchy's inequality.

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